

Mathematics (MEI)

Advanced GCE 4767

Statistics 2

Mark Scheme for June 2010

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of pupils of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

© OCR 2010

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

Question 1

<p>(i)</p> <table border="1" data-bbox="188 383 901 629"> <tbody> <tr><td>x</td><td>6</td><td>17</td><td>9</td><td>20</td><td>13</td><td>15</td><td>11</td><td>14</td></tr> <tr><td>y</td><td>6</td><td>13</td><td>10</td><td>11</td><td>9</td><td>7</td><td>12</td><td>15</td></tr> <tr><td>Rank x</td><td>8</td><td>2</td><td>7</td><td>1</td><td>5</td><td>3</td><td>6</td><td>4</td></tr> <tr><td>Rank y</td><td>8</td><td>2</td><td>5</td><td>4</td><td>6</td><td>7</td><td>3</td><td>1</td></tr> <tr><td>d</td><td>0</td><td>0</td><td>2</td><td>-3</td><td>-1</td><td>-4</td><td>3</td><td>3</td></tr> <tr><td>d^2</td><td>0</td><td>0</td><td>4</td><td>9</td><td>1</td><td>16</td><td>9</td><td>9</td></tr> </tbody> </table> <p>$\Sigma d^2 = 48$</p> $r_s = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)} = 1 - \frac{6 \times 48}{8 \times 63}$ $= 0.429 \text{ (to 3 s.f.) [allow 0.43 to 2 s.f.]}$	x	6	17	9	20	13	15	11	14	y	6	13	10	11	9	7	12	15	Rank x	8	2	7	1	5	3	6	4	Rank y	8	2	5	4	6	7	3	1	d	0	0	2	-3	-1	-4	3	3	d^2	0	0	4	9	1	16	9	9		<p>M1 for attempt at ranking (allow all ranks reversed)</p> <p>M1 for d^2</p> <p>A1 CAO for Σd^2</p> <p>M1 for method for r_s</p> <p>A1 f.t. for $r_s < 1$ NB No ranking scores zero</p>	5
x	6	17	9	20	13	15	11	14																																																	
y	6	13	10	11	9	7	12	15																																																	
Rank x	8	2	7	1	5	3	6	4																																																	
Rank y	8	2	5	4	6	7	3	1																																																	
d	0	0	2	-3	-1	-4	3	3																																																	
d^2	0	0	4	9	1	16	9	9																																																	
<p>(ii)</p> <p>H_0: no association between X and Y in the population H_1: some positive association between X and Y in the population</p> <p>One tail test critical value at 5% level is 0.6429 Since $0.429 < 0.6429$, there is insufficient evidence to reject H_0,</p> <p>i.e. conclude that there is not enough evidence to show positive association between the two judges' scores.</p>		<p>B1 for H_0 B1 for H_1 B1 for population SOI NB $H_0 H_1$ <u>not</u> ρ B1 for ± 0.6429 M1 for sensible comparison with c.v., provided that $r_s < 1$ A1 for conclusion in context f.t. their r_s and sensible cv</p>	3 3																																																						
<p>(iii)</p> <p>A bivariate Normal distribution is required.</p> <p>Scatter diagram.</p> <p>Suitable discussion</p>		<p>B1 G1 labelled axes G1 correct points E1 E1</p>	5																																																						
		TOTAL	16																																																						

Question 2

(i)	Counts have a uniform average rate of occurrence All counts are independent	E1 E1	2
(ii)	Variance = 3.4	B1	1
(iii)	(A) Either $P(X=3) = 0.5584 - 0.3397 = 0.2187$ Or $P(X=3) = e^{-3.4} \frac{3.4^3}{3!} = 0.2186$	M1 for use of tables or calculation A1	2
	(B) Using tables: $P(X \geq 3) = 1 - P(X \leq 2)$ $= 1 - 0.3397$ $= 0.6603$	M1 for $1 - P(X \leq 2)$ M1 correct use of Poisson tables A1	
(iv)	$\lambda = 12 \times 3.4 = 40.8$ $P(X=40) = e^{-40.8} \frac{40.8^{40}}{40!} = 0.0625$	B1 for mean M1 for calculation A1	3
(v)	Mean no. per hour = $12 \times 3.4 = 40.8$ Using Normal approx. to the Poisson, $X \sim N(40.8, 40.8)$ $P(X \geq 40) = P\left(Z > \frac{39.5 - 40.8}{\sqrt{40.8}}\right)$ $= P(Z > -0.2035) = \Phi(0.2035)$ $= 0.5806$	B1 for Normal approx. B1 for correct parameters (SOI) B1 for correct continuity corr. M1 for probability using correct tail A1 CAO (3 s.f.)	5
(vi)	Overall mean = 4.8 $P(X \geq 8) = 1 - P(X \leq 7)$ $= 1 - 0.8867 = 0.1133$	B1 for 4.8 M1 A1	3
		TOTAL	19

Question 3

(i)	<p>(A) $P(X < 65) =$ $P\left(Z < \frac{65-63}{5.2}\right)$ $= P(Z < 0.3846)$ $= \Phi(0.3846) = 0.6497$</p> <p>(B) $P(60 < X < 65) = P\left(\frac{60-63}{5.2} < Z < \frac{65-63}{5.2}\right)$ $= P(-0.5769 < Z < 0.3846)$ $= \Phi(0.3846) - (1 - \Phi(0.5769))$ $= 0.6497 - (1 - 0.7181)$ $= 0.3678$</p>	<p>M1 for standardizing</p> <p>M1 for structure A1 CAO (min 3 s.f.), NB When a candidate's answers suggest that (s)he appears to have neglected to use the difference column of the Normal distribution tables penalise the first occurrence only</p> <p>M1 for standardizing both M1 for correct structure</p> <p>A1 CAO 3s.f.</p>	<p>3</p> <p>3</p>
(ii)	<p>$P(\text{All 5 between 60 and 65})$ $= 0.3678^5 = 0.00673$</p>	<p>M1 A1 FT (min 2sf)</p>	<p>2</p>
(iii)	<p>From tables $\Phi^{-1}(0.95) = 1.645$</p> $\frac{k-63}{5.2} = -1.645$ $x = 63 - 5.2 \times 1.645 = 54.45 \text{ mins}$	<p>B1 for ± 1.645 seen M1 for correct equation in k</p> <p>A1 CAO</p>	<p>3</p>
(iv)	<p>$H_0: \mu = 63$ minutes; $H_1: \mu < 63$ minutes. Where μ denotes the population mean time on the new course.</p> $\text{Test statistic} = \frac{61.7-63}{5.2/\sqrt{15}} = \frac{-1.3}{1.3426}$ $= -0.968$ <p>5% level 1 tailed critical value of $z = 1.645$ $-0.968 > -1.645$ so not significant. There is not sufficient evidence to reject H_0</p> <p>There is insufficient evidence to conclude that the new course results in lower times.</p>	<p>B1 for use of 63 B1 for both correct B1 for definition of μ</p> <p>M1 must include $\sqrt{15}$</p> <p>A1</p> <p>B1 for ± 1.645 M1 for sensible comparison leading to a conclusion</p> <p>A1 FT for correct conclusion in words in context</p>	<p>3</p> <p>5</p>
			<p>19</p>

Question 4

<p>(i)</p>	<p>H_0: no association between category of runner and type of running; H_1: some association between category of runner and type of running;</p> <table border="1" data-bbox="172 465 874 622"> <thead> <tr> <th>EXPECTED</th> <th>Junior</th> <th>Senior</th> <th>Veteran</th> </tr> </thead> <tbody> <tr> <td>Track</td> <td>5.13</td> <td>7.84</td> <td>6.03</td> </tr> <tr> <td>Road</td> <td>6.48</td> <td>9.90</td> <td>7.62</td> </tr> <tr> <td>Both</td> <td>5.40</td> <td>8.25</td> <td>6.35</td> </tr> </tbody> </table> <table border="1" data-bbox="172 694 874 851"> <thead> <tr> <th>CONTRIBUTN</th> <th>Junior</th> <th>Senior</th> <th>Veteran</th> </tr> </thead> <tbody> <tr> <td>Track</td> <td>2.9257</td> <td>0.0032</td> <td>2.6949</td> </tr> <tr> <td>Road</td> <td>0.9468</td> <td>0.3663</td> <td>2.5190</td> </tr> <tr> <td>Both</td> <td>0.3615</td> <td>0.3694</td> <td>0.0192</td> </tr> </tbody> </table> <p>$\chi^2 = 10.21$</p> <p>Refer to χ_4^2</p> <p>Critical value at 5% level = 9.488</p> <p>Result is significant</p> <p>There is evidence to suggest that there is some association between category of runner and type of running. NB if H_0 H_1 reversed, or 'correlation' mentioned, do not award first B1 or final E1</p>	EXPECTED	Junior	Senior	Veteran	Track	5.13	7.84	6.03	Road	6.48	9.90	7.62	Both	5.40	8.25	6.35	CONTRIBUTN	Junior	Senior	Veteran	Track	2.9257	0.0032	2.6949	Road	0.9468	0.3663	2.5190	Both	0.3615	0.3694	0.0192	<p>B1</p> <p>M1 A2 for expected values (to 2 dp) (allow A1 for at least one row or column correct)</p> <p>M1 for valid attempt at $(O-E)^2/E$ A1 for all correct <small>NB These M1/A1 marks cannot be implied by a correct final value of χ^2</small></p> <p>M1 for summation A1 for χ^2</p> <p>B1 for 4 deg of f B1 CAO for cv B1 FT their 'sensible' χ^2</p> <p>E1 must be consistent with their χ^2</p>	<p>1</p> <p>7</p> <p>4</p>
EXPECTED	Junior	Senior	Veteran																																
Track	5.13	7.84	6.03																																
Road	6.48	9.90	7.62																																
Both	5.40	8.25	6.35																																
CONTRIBUTN	Junior	Senior	Veteran																																
Track	2.9257	0.0032	2.6949																																
Road	0.9468	0.3663	2.5190																																
Both	0.3615	0.3694	0.0192																																
<p>(ii)</p>	<ul style="list-style-type: none"> • Juniors appear be track runners more often than expected and road less often than expected. • Seniors tend to be as expected in all three categories of running. • Veterans tend to be road runners more than expected and track runners less than expected. 	<p>E1 E1</p> <p>E1 E1</p> <p>E1 E1</p>	<p>6</p>																																
		<p>TOTAL</p>	<p>18</p>																																

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)

Telephone: 01223 553998

Facsimile: 01223 552627

Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity



OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553